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Portfolio Strategy

Limits of Leverage

Leverage can significantly expand the range of portfolio choices. However, in practice, multiple constraints exist that limit the use of leverage, the nature of the assets that can be leveraged, and the acceptable levels of total portfolio and asset-specific risks.

Such constraints may force the available efficient frontier to shrink to a surprisingly narrow set of portfolios — ones that fall along a single, modestly sloped line located in the middle of the risk/return space.

Any form of leverage is also subject to a number of special concerns: vulnerability to changing financing costs, unanticipated capital calls, illiquidity spirals, etc.

On the one hand, the perceived advantages of leverage can induce a temptation for excessive risk-taking. On the other hand, one must be sensitive to the fact that pragmatic issues and cautionary concerns may keep many funds from even considering the use of leverage. In all cases, it can be enlightening — even for funds that are currently prohibited from using leverage — to envision how other investors that do use leverage can influence the common investment landscape.

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Limits of Leverage

We thank Dr Stanley Kogelman, who is a consultant (not a member of Morgan Stanley's Research department), for his important contributions to the development of the mathematics and the research in this report. (Unless otherwise indicated, his views are his own and may differ from the views of the Morgan Stanley Research department and from the views of others within Morgan Stanley).

Summary & Conclusions

In a previous paper [1], we focused on the relationship between bond duration, Sharpe ratio (SR) and leverage in creating an equity/bond portfolio with the greatest possible return/risk ratio. It was shown that this "tangent" portfolio is determined by the equity and bond SRs.

By leveraging this tangent portfolio to reach a volatility target, we illustrated the theoretical benefits of leverage. Any potential gains must, of course, be weighed against an array of risks, many of which are not easily quantified. Even the core risk/return assumptions that are the foundation of any leverage strategy must be regarded as estimates, and it should be recognized that leverage will exacerbate the impact of any realized deviation from these core assumptions.

There are also a myriad of other leverage-specific risks, including rollover financing costs, yield curve reshaping, unanticipated bouts of inflation/deflation and, perhaps most importantly, spiking capital calls.

In this report, we show that 1) separate leverage constraints on either equity or bonds are theoretically equivalent to some total portfolio leverage constraint, and 2) conversely, any total portfolio leverage constraint may be realized with various combinations of bond leverage and equity leverage.

We further show how a variety of practical risk constraints for either leveraged or unleveraged portfolios can lead to a rather limited set of investment choices that fall along a single line segment. Along this segment, the "local" SR (i.e. the marginal gain achieved from taking on increased risk) typically is well below either the equity or bond SR. Nevertheless, most institutional portfolios have a volatility (around 10%) that places them at roughly the mid-point of this frontier segment. From this observation, we might infer that the basic volatility constraint plays a more dominant role in portfolio selection than theoretical criteria based on risk/return trade-offs.

Many other authors have analyzed the opportunities, risks and limitations of leverage from a variety of perspectives [2 –6]. However, in this report, we focus more narrowly on how leverage constraints can reshape and redefine the basic two-asset frontier.

The Standard Unconstrained Frontier

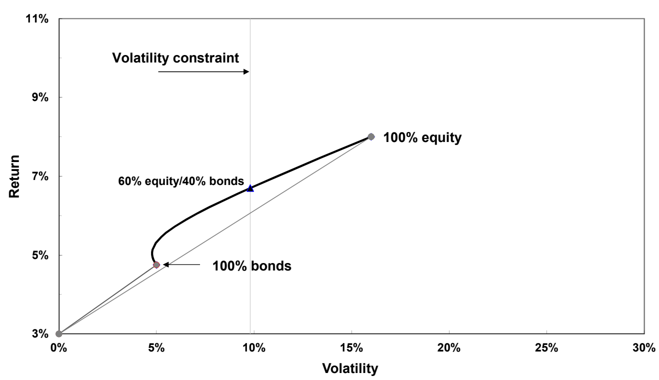
The traditional unlevered stock/bond efficient frontier plots return against volatility risk. Each frontier point represents an asset allocation with the maximum expected return for a given level of risk (volatility). Bonds of low to intermediate duration form the lower-risk asset class. Equity-like assets generally offer higher risk and higher return than bonds. The historical correlation between equity and bonds is modestly positive at times and modestly negative at other times.

For simplicity, we focus on a base-case equity/bond model with zero correlation, five-year duration bonds, a bond spread over cash of 1.75%, interest rate volatility of 1%, an equity risk premium of 5%, and equity volatility of 16%.

At the outset, the diversification benefit of including assets with low correlation is realized when small amounts of bonds are replaced with equity. This asset exchange increases return, decreases volatility and generates the classic "bullet-nose" that characterizes most efficient frontier diagrams (see Exhibit 1). As the equity allocation grows, the volatility of equity soon overwhelms the diversification benefits, the efficient frontier begins to flatten and then remains positively sloped as the allocation moves toward the 100% equity point.

Exhibit 1

Efficient Frontier with Bond and Equity Leverage Lines



Source: Morgan Stanley Research

In Exhibit 1 we also highlight the more or less standard "60/40" portfolio. With our assumed zero correlation, the portfolio has a beta sensitivity of exactly 0.60, corresponding to the 60% equity weight. It is interesting to note that this 0.60 beta sensitivity characterizes a wide range of multi-asset institutional portfolios, including many with very low explicit allocations to equity [7]. Indeed, one rarely sees portfolios with betas that are either much higher or much lower than 0.6. This widespread presence of the 0.6 beta suggests that some common decision process might be at work, at least implicitly.

With standard volatility assumptions, the 0.6 beta leads to a portfolio volatility of about 10%, for both the simple 60/40 portfolio and (somewhat more approximately) for typical diversified funds. In turn, this 10% volatility corresponds to about a 10% probability of a -13% loss. It is tempting to suggest that most investors would find this prospect sufficiently distasteful that they would structure their portfolios to avoid breaching this risk limit. To obtain the best possible return given this risk constraint, funds would then push their beta right up to this 0.6 limit. Such a hypothetical decision process offers a possible explanation for why so many portfolio beta values are clustered around the 0.6 value.

Leverage Lines

Exhibit 2 displays "leverage lines" for bonds and equity. Each of these lines emanates from the 100% cash point and passes through the corresponding 100% bond point or 100% equity point. Portfolios along the leverage lines include varying proportions of cash. For example, the leftmost point of the bond leverage line represents 0% bonds/100% cash. The rightmost point represents a leveraged 150% bond holding and a 50% cash borrowing.

The somewhat higher slope of the bond leverage line, in comparison to the equity leverage line, reflects our illustrative base-case assumption that bonds have a slightly higher Sharpe ratio:

SR(Bonds) = 1.75%/5% = 0.35

SR(Equity) = 5%/16% = 0.3125.

Our findings will only change slightly if SR(equity) is somewhat greater than SR(bonds).

The leverage lines theoretically can be continued indefinitely as a hypothetical investor takes on ever more risk in order to capture higher returns. We terminate these lines at 1.5x leverage to reflect one specific leverage constraint. If the initial portfolio value is $100, a 1.5x leverage multiple implies a borrowing of $50, with the proceeds invested in bonds. The total long position is then $150.

In this simplified leverage model, we assume no incremental spread of borrowing costs over cash, i.e., the investor can borrow at the riskless 3% cash rate. In practice, all investors must pay some financing spread. For large institutions, this spread may be only a few basis points. Borrowing costs for smaller institutions and individual investors can be far higher.

Leveraging the Tangent Portfolio

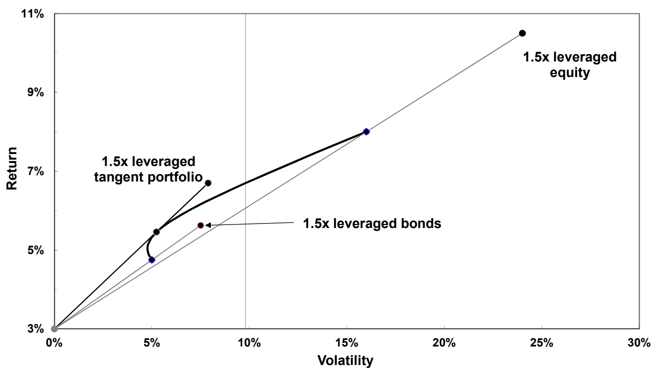
It can be shown [8-9] that if the correlation is zero, the optimal SR is the square root of the sum of the squares of the bond and equity SRs. Thus, the optimal SR is generally greater than either of the component SRs.

For frontiers with the typical shape of Exhibit 1, the optimal risk/return trade-off (that is, the point with the highest SR) is found at a "tangent portfolio" that lies above and just a bit to the right of the minimum volatility point (see Exhibit 2).

Leverage can theoretically be used to develop expected returns higher than those available in the standard efficient frontier. The highest leveraged return for a given risk is achieved by constructing a leverage line that passes through the tangent portfolio.

Exhibit 2

Portfolios Along the Tangent Line Reflect the Optimal Gains from Leverage



Source: Morgan Stanley Research

In Exhibit 2, the tangent portfolio is comprised of 22% equity/78% five-year duration bonds and has an SR of 0.47. This high concentration of the lower-volatility bond asset is a general characteristic of most efficient frontiers and is reflective of the shape considerations discussed earlier. If the bond portfolio has duration less than the assumed five years, the tangent point would lie further to the left, have a steeper slope and include even more bonds.

Because of the tangent portfolio's concentrated position in the lower-volatility bond asset, this portfolio has a relatively modest return (5.5%) and low volatility (5.2%). To attain minimally required levels of return, investors must either leverage the tangent portfolio or move to higher-volatility (but less optimal) positions along the long-only frontier.

Return Enhancement vs. Risk Reduction

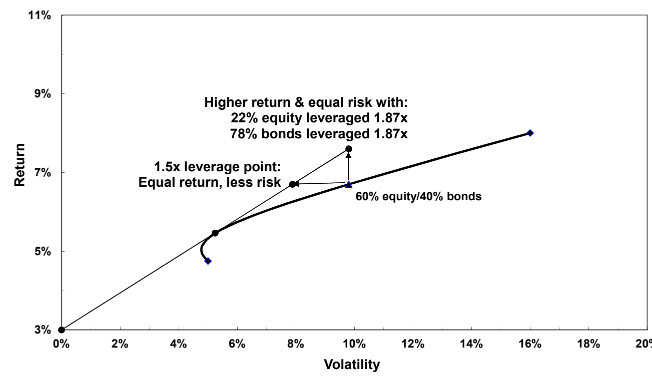
In theory, leverage can be used to enhance return or to reduce risk. In practice, leverage should be applied cautiously and only undertaken with full recognition of both the transparent and the subtle risks.

To illustrate the theoretical advantages of leverage, we focus on the standard 60% equity/40% bond portfolio. From our base-case assumptions, it follows that the volatility and return of this portfolio are 9.8% and 6.7%, respectively.

Return enhancement is achieved if the tangent portfolio is leveraged 1.87x. The resulting portfolio has the same 9.8% volatility, but with a 90 basis point greater expected return (Exhibit 3). Risk reduction is achieved when the optimal portfolio is leveraged 1.5x. The resulting portfolios will have the same 6.7% expected return as the 60/40 portfolio, but with a reduced volatility of only 7.9%.

Exhibit 3

"Leverage" Can Either Add Return or Reduce Risk Relative to 60% Equity/40% Bond Portfolio



Source: Morgan Stanley Research

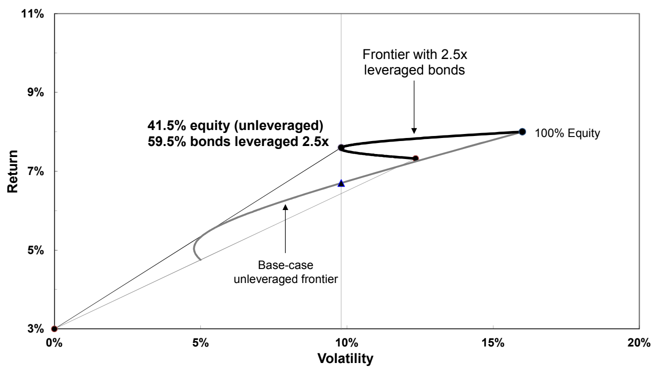
At the maximum return point in Exhibit 3, the portfolio consists of 78% bonds leveraged 1.87x and 22% equity leveraged 1.87x. In practice, instead of applying uniform leverage, it may be more convenient (or comfortable) to leverage only bonds and not equity. It turns out that this same maximum return point could be obtained by leveraging bonds by a 2.5x factor and then combining them with 41.5% unleveraged equity (Exhibit 4).

Exhibit 4 also implicitly illustrates the more general result that every point between the tangent line and the bond leverage line can be reached by mixing sufficiently leveraged bonds with unleveraged equity. Doing so may, however, require unacceptably high levels of bond leverage.

In Exhibit 5, we show an efficient frontier that represents all possible combinations of unleveraged equity and 1.5x leveraged bonds. In this case, the return advantages of leverage are markedly diminished. At the 9.8% volatility-matching point, the new frontier is above the unleveraged frontier, but the return is far below that of the optimal leveraged portfolio.

Exhibit 4

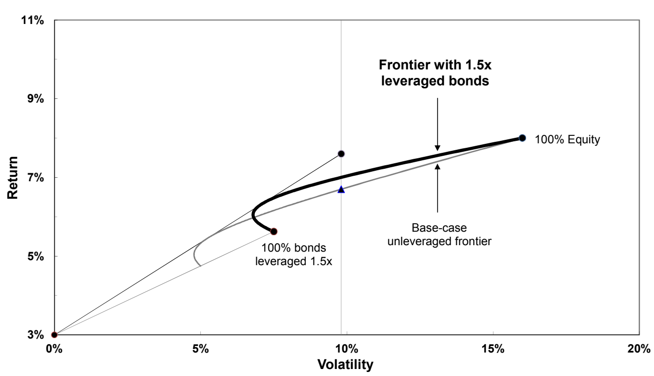
Efficient Frontier with No Equity Leverage and 2.5× Leveraged Bonds



Source: Morgan Stanley Research

Exhibit 5

Efficient Frontier with No Equity Leverage and 1.5× Leveraged Bonds



Source: Morgan Stanley Research

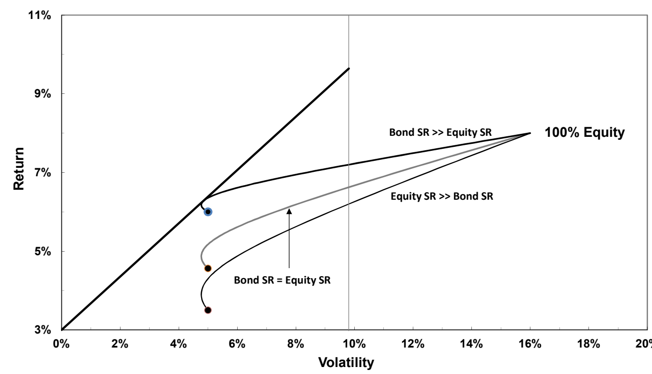
Sharpe Ratios

We now turn our attention to the impact of correlation, SR and volatility on optimal leverage. In the zero-correlation base case, the tangent slope is solely dependent on the component SRs. However, the location of the tangent portfolio itself will depend on the respective volatilities (see Exhibit 6).

If the bond component has a higher SR, the tangency point moves toward bonds and the steep part of the efficient frontier, and the potential leverage benefit increases dramatically. In contrast, if the bond component has a lower SR, the location of the tangency point will move toward equity and the leverage benefit will become quite limited.

Exhibit 6

Greater Leverage Benefits with Higher Bond SRs



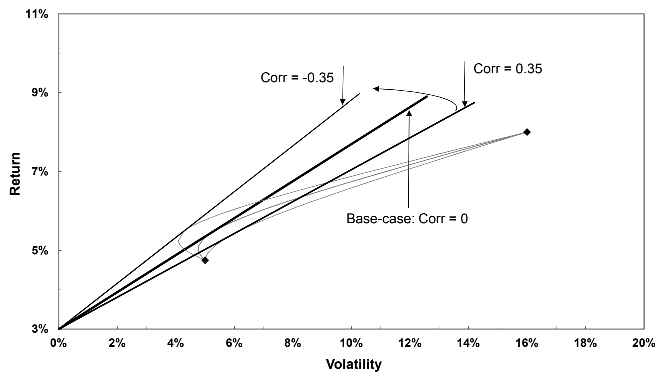
Source: Morgan Stanley Research

In Exhibits 7 and 8, we compare the tangent lines corresponding to higher and lower correlations. In the base case with SR (Bonds) at 0.35, the efficient frontier begins to flatten as the correlation becomes increasingly positive and diversification benefits decrease. The slope of the tangent line then decreases and the efficient frontier ultimately becomes a straight line from the bond point to the equity point. The tangent line will then pass directly through the 100% bond point and its slope will equal the 0.35 SR of bonds.

As the correlation becomes more negative, the “bullet nose” of the frontier is pulled to the left and the tangent slope increases dramatically. For the base case SRs, the tangent slope reaches 1.5 at a correlation of -0.9, and increases without limit as the correlation approaches -1.0.

Exhibit 7

Negative Correlation Implies Higher Optimal SR; Positive Correlation Implies Lower Optimal SR



Source: Morgan Stanley Research

Moving from the base case with SR(B) = 0.35, the last two columns in Exhibit 8 reflect situations in which the underlying yield curve is either flat (SR(B) = 0) or negatively sloped (SR(B) = -0.10). In both cases, if the correlation is zero or greater, the tangent will pass directly through the equity point and its slope will equal the 0.31 equity SR. Thus, it would be counterproductive to include such bonds in the investment portfolio.

In contrast, extreme negative correlations can make bonds attractive even if their SR is negative. For example, with SR(B) = -0.1, a correlation of -0.9 will lead to a high tangent slope and substantial leverage opportunities.

Exhibit 8

Maximum Portfolio SR (Tangent Slope) for a Range of Correlations and Bond SRs (with SR(E) = 0.31)

Correlation	Base Case SR(B) = 0.35	SR(B) = 0	SR(B) = -0.10
0.90	0.35	0.31	0.31
0.35	0.40	0.31	0.31
0.00	*0.47*	*0.31*	*0.31*
-0.35	0.58	0.33	0.31
-0.90	1.48	0.72	0.52

Risks of Leverage

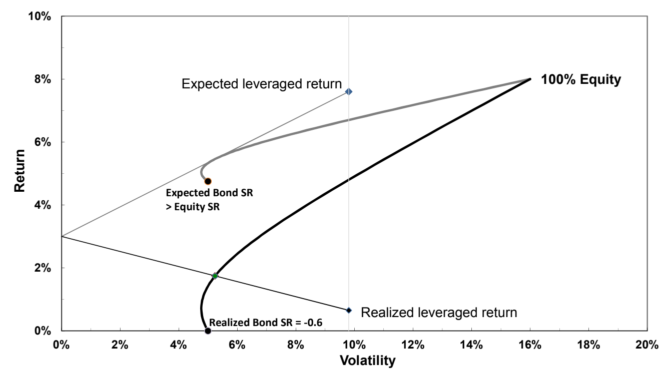
Leverage magnifies the impact of any differential between the expected return and the realized return. More specifically, the return potential of any leveraged bond strategy will be achieved only if the realized excess bond return over the underlying investment horizon turns out to be close to the anticipated value. A rising yield curve or yield curve re-shaping that occurs over the holding period may significantly reduce the assumed bond return premium and /or adversely impact rollover financing costs.

Exhibit 9 illustrates the impact of a realized excess bond return that falls well below the expected value. The negatively sloped realized leverage line is indicative of how leverage can exacerbate weak (or negative) bond returns.

In addition to these interest rate and rollover risks, leveraged portfolios are particularly sensitive to market movements, margin calls and liquidity squeezes.

Exhibit 9

Leveraging Effect with Lower Realized Bond Return



Source: Morgan Stanley Research

Asset-Specific Leverage

In Exhibits 3 and 4, we saw that the maximum return point corresponding to 9.8% volatility could be reached by either 1) 1.87 uniform leverage of the tangent portfolio, or 2) asset-specific 2.5x bond-only leverage.

At first, it may seem odd that the same maximum return point could be reached by two such different leveraging procedures. It turns out that both the uniformly-leveraged tangent portfolio and the portfolio with bond-only leverage actually have the exact same underlying sensitivities to interest rates (dollar-duration) and equity market moves (portfolio beta).

The beta of the leveraged tangent portfolio is 0.41 (21.8% equity x 1.87 leverage). For the optimal portfolio with 2.47x leveraged bonds, the equity allocation is 41%. Since no leverage is applied to the equity component, the beta is 0.41. It can similarly be shown that both portfolios have the same dollar duration.

This observation has several important implications and generalizations (see Appendix II):

1) the same equity risk and dollar-duration exposures generally can be achieved in a variety of ways

2) any feasible return/risk point can be reached by either uniform leverage or by various mixtures of asset-specific leverage

3) all leveraging procedures leading to a given portfolio beta and dollar-duration necessarily have the same total leverage

4) a given portfolio risk/return point that lies to the right of the tangent line and above the efficient frontier can have only one "efficient" beta and dollar-duration.

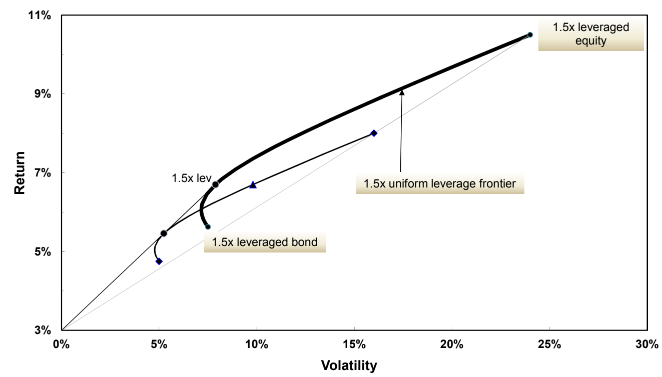
The Uniformly Leveraged Frontier

In the earlier sections of this paper, we focused on movement along the tangent line, reflecting the gains that leverage can provide, at least in theory. Such movements entailed uniformly leveraging the bond and equity components of the 22% equity/78% bonds tangent portfolio. For example, in Exhibit 2, we highlighted the point corresponding to the 1.5x leveraged bond and equity positions. With such uniform leverage, the proportion of equity and bonds remains the same as in the initial long-only portfolio.

In Exhibit 10, we illustrate the efficient frontier that results when the same 1.5x uniform leverage is applied, not just to the tangent portfolio, but to every bond/equity portfolio along the original long-only frontier. It can also be shown that precisely the same leveraged frontier would be generated by just leveraging bonds by 1.5x, equities by 1.5x, and then creating combinations of these two leveraged assets. The new frontier will have the same tangent line as the unleveraged frontier. This tangent-sharing follows from the fact that leverage does not change SR(B), SR(E) or the optimal SR.

Exhibit 10

A Uniformly 1.5x Leveraged Efficient Frontier



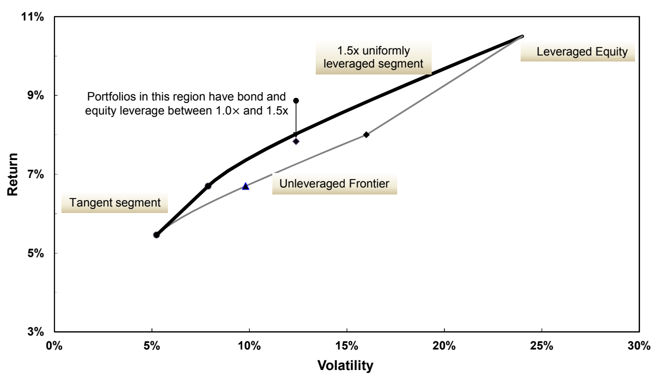
Source: Morgan Stanley Research

If we now impose a maximum 1.5x total leverage constraint, we will trace out a new efficient frontier (Exhibit 11) that incorporates part of the tangent line and part of the 1.5x leverage frontier. Starting at the tangent point on the unleveraged frontier, we steadily increase leverage along the tangent line until we reach 1.5x. Any further movement along the tangent line will increase leverage beyond this limiting value, so we move off the tangent line and follow the 1.5x leveraged frontier until we reach the point consisting of 100% 1.5x leveraged equity.

The region below this multi-segment frontier incorporates all portfolios with less than 1.5x *total* leverage. For example, a portfolio comprised of 75% unleveraged equity and 25% bonds leveraged 2.0x (the indicated point in Exhibit 11) would fall within this region. The total leverage of this portfolio is 1.25x (75% of 1.0x leverage and 25% of 2.0x leverage).

Exhibit 11

With Total Leverage Limited to 1.5x, New Frontier Follows the Tangent Line and then Shifts to the Uniform Leverage Frontier



Source: Morgan Stanley Research

Beta Constraints and Local Slope of a Single-Segment Frontier

The previous sections focused only on leverage constraints. Most investors are also likely to impose additional constraints such as overall beta and duration limits. To explore the impact of beta constraints, Exhibit 12 superimposes limits of 0.40 < β < 0.75 on the 1.5x leverage-constrained frontier of Exhibit 10.

This further constrained frontier has little curvature and only modest "local" slope. The implication of ending up with such a flat constrained frontier is that portfolio choice becomes considerably more limited. As we range from the minimum to maximum points on B, the volatility ranges from 7.8% to 12.2%, and return ranges from 6.6% to 7.4%.

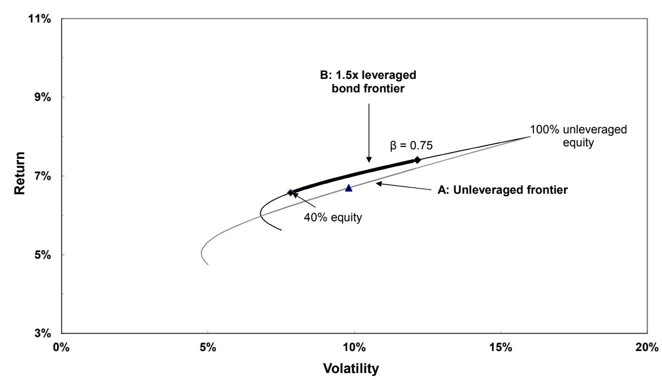
The local slope along this segment is only 0.18, far less than the bond, equity or tangent SRs. Thus, one may question whether this reduced risk-taking incentive justifies driving the risk to higher points on the segment or even up to the 10% volatility limit so commonly observed in institutional portfolios.

In Exhibit 13, we impose the same 0.40 to 0.75 beta constraints as in Exhibit 11 on the unleveraged frontier, A. In this case, the constrained frontier is also relatively flat, but the local slope of A is 0.24, somewhat higher than for B. Volatility ranges from 7.1% to 12.1% and return ranges from 6.0% to 7.2%.

With beta constraints, the return difference between the lower unleveraged segment A and the upper leveraged segment B is rather narrow, implying relatively limited incremental gain from leverage. Thus, the combination of leverage constraints and maximum beta constraints significantly limits the maximum return advantage of leverage. To increase the return advantage we would have to allow total leverage to expand beyond the 1.5x limit.

Exhibit 12

Additional Constraints Lead to a "Straight Line" Efficient Frontier: Min Equity = 40%, Max Beta = 0.75



Source: Morgan Stanley Research

In contrast, as we move toward the lower end of the range of admissible betas, the relative gains from leverage (B versus A) increase. In essence there is relatively little to be gained from taking on both high beta-risk and leverage.

More generally, when (approximately straight) line segments form the feasible frontier, the portfolio choice becomes more dependent on the nature of the investor's decision criteria:

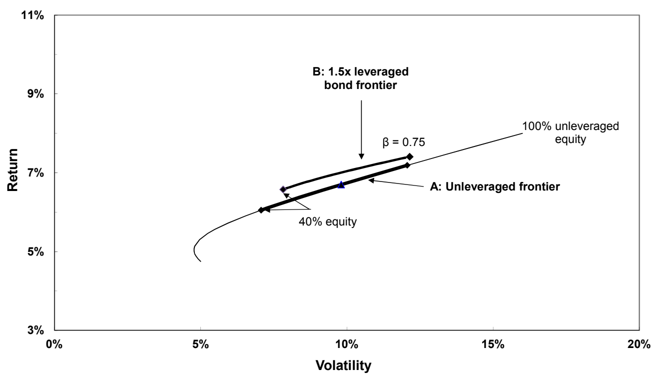
1) The theoretically standard mean/variance optimization approach will lead to some interior point on the line segment that is "utility maximizing"

2) With some required minimum level of incremental return and incremental risk, the choice will be driven toward one of the two endpoints of the line segments

3) The portfolio choice may be more simply determined by projecting either a vertical line at the maximum risk point (as in Exhibit 2) or horizontal line (at the minimum return point through the feasible line segment.

Exhibit 13

Beta-Constrained Unleveraged and Leverage-Limited Frontiers: Min Equity = 40%, Max Beta = 0.75



Source: Morgan Stanley Research

In practice, many portfolio decisions seem to be determined by these more basic minimum return or maximum risk targets without consideration of the local slope. This observation may, in part, explain the very narrow range of observed portfolio betas.

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Technical Appendix I: Sharpe Ratio Maximization

For a portfolio comprised of assets B and E, we derive formulas for the maximum Sharpe Ratio (SR) and corresponding asset weights. The expected excess return relative to the risk-free rate, volatility and SR are symbolized by μ_B, μ_E, σ_B, σ_E, SR_B, SR_E, respectively.

In an earlier paper, we assumed the correlation (ρ) between B and E was zero. In this paper, we extend that result to the non-zero correlation case.

The portfolio return μ_P and portfolio variance σ_P^2 are given by the equations below, assuming the weights of B and E are ω and 1 – ω.

$$\mu_P(\omega) = \omega\mu_B + (1-\omega)\mu_E$$

$$\mu_P(\omega) = \omega\sigma_B SR_B + (1-\omega)\sigma_E SR_E \qquad (1)$$

$$\mu_P(\omega) = \omega k_1 + \sigma_E SR_E \qquad (2)$$

$$k_1 = \sigma_B SR_B - \sigma_E SR_E$$

$$SR_B = \mu_B/\sigma_B, \; SR_E = \mu_E/\sigma_E$$

$$SR(\omega) = \frac{\mu_P(\omega)}{\sigma_P(\omega)} \qquad (3)$$

$$\sigma_P^2(\omega) = (\omega\sigma_B)^2 + [(1-\omega)\sigma_E]^2 + 2\omega(1-\omega)\rho\sigma_B\sigma_E \qquad (4)$$

$$\sigma_P^2(\omega) = \omega^2 k_2 - 2\omega\sigma_E k_3 + {\sigma_E}^2 \qquad (5)$$

$$k_2 = \sigma_B^2 + \sigma_E^2 - 2\rho\sigma_B\sigma_E \qquad (6)$$

$$k_3 = \sigma_E - \rho\sigma_B \qquad (7)$$

To maximize SR, set $SR_P^{'}$= 0:

$$SR_P^{'}(\omega) = \frac{\mu_P^{'}(\omega)\sigma_P(\omega) - \mu_P(\omega)\sigma_P^{'}(\omega)}{\sigma_P^2(\omega)} = 0$$

$SR_P^{'}$= 0 if:

$$\sigma_P^2\mu_P^{'} = \sigma_P\sigma_P^{'}\mu_P \qquad (8)$$

From 2:

$$\mu_P^{'} = k_1 \qquad (9)$$

From 5:

$$\sigma_P\sigma_P^{'} = \omega k_2 - \sigma_E k_3 \qquad (10)$$

Using 2,5,8,9,10 in equation 8:

$$(\omega^2 k_2 - 2\omega\sigma_E k_3 + {\sigma_E}^2)k_1 = (\omega k_2 - \sigma_E k_3)(\omega k_1 + \sigma_E SR_E)$$

Rearranging:

$${\sigma_E}^2(k_1 + k_3 SR_E) = \omega\sigma_E(k_2 SR_E + k_1 k_3)$$

Solve for maximizing weight: $\omega^* = \dfrac{\sigma_E(k_1 + k_3 SR_E)}{(k_2 SR_E + k_1 k_3)}$

Since $k_1 + k_3 SR_E = \sigma_B SR_B - \sigma_E SR_E + (\sigma_E - \rho\sigma_B) SR_E$

$= \sigma_B(SR_B - \rho SR_E)$ (11)

and $k_2 SR_E + k_1 k_3 = (\sigma_B^2 + \sigma_E^2 - 2\rho\sigma_B\sigma_E)SR_E + (\sigma_B SR_B - \sigma_E SR_E)(\sigma_E - \rho\sigma_B)$

$= \sigma_B(\sigma B_B SR_E + \sigma_E SR_B) - \rho\sigma_B((\sigma_B SR_B + \sigma_E\, SR_E)$ (12)

$\omega^* = \dfrac{\sigma_E(SR_B - \rho SR_E)}{\mathrm{K}}$ (13)

$1 - \omega^* = \dfrac{\sigma_B(SR_E - \rho SR_B)}{\mathrm{K}}$ (14)

and $\mathrm{K} = (\sigma_B SR_E + \sigma_E SR_B) - \rho(\sigma_B SR_B + \sigma_E\, SR_E)$ (15)

Using 13 - 15 in 2: $\mu_P(\omega^*) = \dfrac{\sigma_E(SR_B - \rho SR_E)\sigma_B SR_B + \sigma_B(SR_E - \rho SR_B)\sigma_E SR_E}{K}$

$= \dfrac{\sigma_B\sigma_E(SR_B^2 + SR_E^2 - 2\rho SR_B SR_E)}{K}$

Using 13 - 15 in 4: $\sigma_P^2(\omega^*) = \dfrac{\sigma_B^2\sigma_E^2[(SR_B - \rho SR_E)^2 + (SR_E - \rho SR_B)^2 + 2\rho(SR_B - \rho SR_E)(SR_E - \rho SR_B)]}{K^2}$

$= \dfrac{\sigma_B^2\sigma_E^2[(1+\rho^2)(SR_B^2 + SR_E^2) - 2\rho SR_B SR_E - 2\rho^2(SR_B^2 + SR_E^2 - \rho SR_B SR_E)]}{K^2}$

$= \dfrac{\sigma_B^2\sigma_E^2[(1-\rho^2)(SR_B^2 + SR_E^2) - 2\rho SR_B SR_E(1-\rho^2)]}{K^2}$

$= \dfrac{\sigma_B^2\sigma_E^2(1-\rho^2)(SR_B^2 + SR_E^2 - 2\rho SR_B SR_E)}{K^2}$

$\sigma_P(\omega^*) = \dfrac{\sigma_B\sigma_E\sqrt{1-\rho^2}\sqrt{SR_B^2 + SR_E^2 - 2\rho SR_B SR_E}}{K}$

provided $\rho < \dfrac{SR_B^2 + SR_E^2}{2SR_B SR_E}$

Substituting in3: $SR(\omega^*) = \dfrac{\sqrt{SR_B^2 + SR_E^2 - 2\rho SR_B SR_E}}{\sqrt{1-\rho^2}}$

If $\rho = 0$: $SR(\omega^*) = \sqrt{SR_B^2 + SR_E^2}$

Technical Appendix II: Leveraged portfolio equivalence

All portfolios that lie above the efficient frontier in Exhibit 2, but below the tangent line reflect various degrees of bond and equity leverage. Within this region any "target" portfolio is characterized its expected excess return μ_T and volatility σ_T. The target portfolio is comprised of assets B and E with expected excess returns (relative to the risk-free rate), volatilities and leverage factors symbolized by μ_B, μ_E, σ_B, σ_E, λ_B, λ_E, respectively. The weights of B and E are ω and 1–ω.

In this appendix, we show that this target portfolio can be characterized by its dollar-duration ($D), portfolio beta (β) and total leverage $\lambda = w\lambda_B + (1-w)\lambda_E$. We also show that there are at most two combinations of $D, β, and λ associated with the target portfolio. Finally, we show that a range of weights and asset-specific leverage factors can combine to provide the same $D, β, λ.

$$\mu_T(\omega) = (\omega\lambda_B)\mu_B + [(1-\omega)\lambda_E]\mu_E$$

Let $x = \omega\lambda_B$

Let $y = (1-\omega)\lambda_E$

$$\mu_T(\omega) = \mu_B x + \mu_E y \quad (1)$$

$$\sigma_T^2(\omega) = (\omega\lambda_B\sigma_B)^2 + [(1-\omega)\lambda_E\sigma_E]^2 + 2\omega(1-\omega)\lambda_B\lambda_E\rho\sigma_B\sigma_E$$

$$\sigma_T^2(\omega) = (\sigma_B x)^2 + (\sigma_E y)^2 + 2\rho\sigma_B\sigma_E xy \quad (2)$$

$$\lambda = x + y \quad (3)$$

The two unknowns, x and y can be determined from the two Equations (1) and (2). Solving (1) for y and substituting in (2) leads to a quadratic equation for x, which necessarily has 0, 1, or 2 solutions. Target portfolios that lie above the tangent line or below the equity leverage line in Exhibit 1 cannot be reached by leveraging efficient frontier portfolios. They are the zero solution points.

Portfolios along the tangent line are one solution points. Portfolios above the efficient frontier and between the bond and equity leverage lines are also one solution points.

The two solution target portfolios are in the region above the bond leverage line, above the efficient frontier and below the tangent line. This illustrated in the example and exhibit that follows.

We illustrate the above with an example based on the within-region portfolio indicated in Exhibit 11.

$$\mu_T(\omega) = 7.83\% \quad (4)$$

$$\sigma_T(\omega) = 12.39\% \quad (5)$$

We will obtain a general expression for x and y with our base assumption of zero correlation. We solve (1) for y in terms of x and substitute the result in (2)

$$\mathrm{y} = (\mu_T - \mu_B x)/\mu_E \quad (6)$$

$$\sigma_T^2 = (\sigma_B x)^2 + (\sigma_E)^2[(\mu_T - \mu_B x)/\mu_E]^2$$

Rearranging terms above

$$0 = ax^2 + bx + c \tag{7}$$

$$a = \left[\sigma_B^2 + \left(\frac{\sigma_E \mu_B}{\mu_E}\right)^2\right]$$

$$b = -2\mu_B \mu_T (\sigma_E/\mu_E)^2$$

$$c = \left[\left(\frac{\sigma_E \mu_T}{\mu_E}\right)^2 - \sigma_T^2\right]$$

Using our base assumptions along with (4) and (5)

$$a = 0.56\%$$

$$b = -1.73\%$$

$$c = 0.85\%$$

Using the above values in (7) and applying the quadratic formula

$$x = \frac{1.7313\% \mp 1.0353\%}{1.1272\%}$$

$$x_1 = 0.6175$$

From (6) $$y_1 = 0.7500$$

From (3) $$\lambda_1 = 1.3675$$

$$x_2 = 2.4544$$

$$y_2 = 0.1071$$

$$\lambda_2 = 2.5615$$

Each of the two solutions defines a unique total leverage. In most cases, all other things being equal, one would prefer less leverage so the solution x_1, y_1 would typically be chosen. We now delve more deeply into the meaning of x and y.

If the second asset, E, is equity, then the expression $(1-\omega)\lambda_E$ represents the portfolio β and, therefore, so does y.

If the first asset, B, is bonds, then the portfolio \$D = $\omega\lambda_B$D = xD, where D is the duration of the bond component. Thus, x and y fully determine the interest rate and equity risk of the portfolio.

Since there are exactly two solutions to (7) for the given target risk and return, there are precisely two possibilities for the combination of leverage, interest rate risk and equity risk that lead to the target portfolio characteristics.

However, there are a wide range of choices for the weights and leverage of the bond and equity components that give rise to the same λ, \$D and portfolio β.

For example, there is always a portfolio that, with the right magnitude of uniform leverage, will achieve the targeted risk/return. In the case of uniform leverage, bond leverage, equity leverage and total leverage are the same. The uniform leverage bond weights corresponding to solutions 1 and 2 are calculated as indicated below

$$\omega = x/\lambda$$

$$\omega_1 = x_1/\lambda_1$$

$$= 0.6175/1.3675$$

Bond weight 1 = 45%

Equity weight 1 = 55%

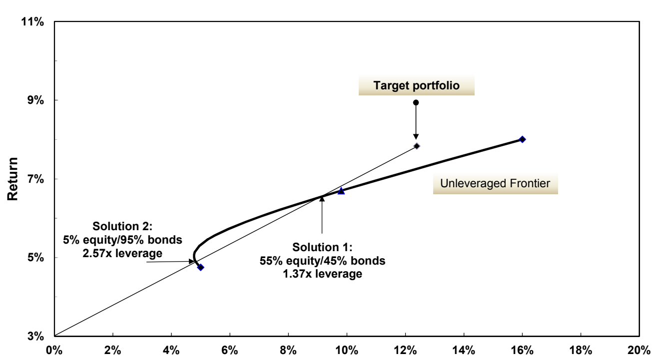
Bond weight 2 = 95%

Equity weight 2 = 5%

Both the above portfolios are on the unleveraged frontier (see Exhibit 14). All leveraged versions of these portfolios lie on the same uniform leverage line passing through the target point. The first portfolio would generally be preferred because it lies on the upper portion of the efficient frontier and requires less leverage to reach the target point. The second portfolio lies on the lower “inefficient” portion of the frontier and requires much higher leverage to reach the target risk/return point.

The same target point could also be reached by means of bond-only leverage. In this case, the equity allocation = 75% ($y_1/\lambda_E = y_1$) the bond allocation = 25%, and the bond leverage = 2.47x (x_1/w_1).

Exhibit 14
Two Distinct Portfolios that, When Leveraged, Produce the Same Risk and Return



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